Optimization B – QEM/MMEF
Université Paris 1 Panthéon–Sorbonne

November 8, 2012
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Midterm Exam (2 hours)

Course Questions (30 points).

1. Give the characterization of non-empty convex sets in $\mathbb{R}$.

2. Let $A = \{a^1, ..., a^j, ..., a^m\}$ be a finite set of points of $\mathbb{R}^n$, $K(A)$ denotes the convex conic hull of A. Give the definition of $K(A)$. Write the complete characterization of the elements of $K(A)$.

3. Consider $C \subseteq \mathbb{R}^n$ and the following maximization problem.

$$(P) \qquad \max \ f(x) \quad \text{subject to } x \in C$$

Give the definitions of $val(P)$ and $Sol(P)$.

4. Give the definition a concave function.

5. Give the definition and the characterization of a quasi-concave function (the function is not necessarily differentiable).

Exercise 1 (35 points). Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \max \{m \in \mathbb{Z} \colon m \leq x\}$$

and the function $\phi : \mathbb{R} \to \mathbb{R}$ defined by $\phi(x) = |x|$.

1. Draw the graph of f and prove that f is quasi-concave on $\mathbb{R}$.

2. We remind that the function $\phi \circ f$ is defined by $(\phi \circ f)(x) = \phi(f(x))$ for every $x \in \mathbb{R}$. Draw the graph of $\phi \circ f$ and show that $\phi \circ f$ is not quasi-concave on $\mathbb{R}$.

From now on, (P) denotes the problem given above and (P_1) denotes the following problem.

$$(P_1) \qquad \max \ \phi(f(x)) \quad \text{subject to } x \in C$$

3. Take $C = [0, 2[$ and determine $val(P)$, $Sol(P)$, $\phi(val(P))$, $val(P_1)$ and $Sol(P_1)$.

4. Same questions for $C =]-2, 2[$.

Exercise 2 (15 points).

1. Give the definition a pseudo-concave function.

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1, x_2) = x_1 x_2$.

2. Show that f is not pseudo-concave on $\mathbb{R}^2$.

3. The function f is not concave on $\mathbb{R}^2$. Why?

Exercise 3 (20 points). Give the statement of the Weierstrass Theorem and prove it.